Eigenvalues, Laplacian eigenvalues, and Hamiltonian connectivity of graphs

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Abstract

Using the eigenvalues or Laplacian eigenvalues of graphs, we present sufficient conditions for Hamiltonian connectivity of graphs.

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1. Introduction

We consider only finite undirected graphs without loops or multiple edges. Notation and terminology not defined here follow that in [1]. For a graph $G = (V, E)$, $n := |V|$, $e := |E|$, and $G^c := (V, E^c)$, where $E^c := \{xy : x \in V, y \in V, x \neq y, xy \notin E\}$. The degree of vertex v_i is denoted by d_i. A graph is Hamiltonian-connected if each pair of vertices in the graph is joined by a Hamiltonian path. The k-closure of a graph G, denoted $\mathrm{cl}_k(G)$, is a graph obtained from G by recursively joining two nonadjacent vertices such that their degree sum is at least k. We use $C(n, r)$ to denote the number of r-combinations of a set with n distinct elements. The graph $K_{n-1} + v$ is defined as a graph that consists of a complete graph of order $n - 1$ together with an isolated vertex v. The graph $K_{n-1}+e$ is defined as a graph that consists of a complete graph of order $n - 1$ together with a pendent edge e. For each k, where $k = 1, 2$, or 3, Q_k is defined as a graph obtained by joining k vertices of the complete graph K_{n-k} to each of k-independent vertices. Notice that $Q_1 = K_{n-1} + e$. The unique graph Q of order n with $C(n - 1, 2) + 2$ edges is obtained by

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