Portfolio value-at-risk by Bayesian conditional EVT-copula models: taking an Asian index portfolio for example

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Abstract
Portfolio VaR (Value-at-Risk) is an important but difficult issue in risk management. Traditional approaches deal with single asset and usually fail to accurately estimate the threshold value under given probability level. This study proposed a new model which estimates portfolio VaR (Value-at-Risk) using a Bayesian conditional EVT (extreme value theory)-Copula based approach. First, GJR-GARCH model is used to model the time structure of each asset. Second, EVT is employed for modeling the residuals after GJR-GARCH. This study constructs the semi-parametric empirical marginal CDF (cumulative distribution

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function) for each residual using a Gaussian kernel estimate for the interior and a generalized Pareto distribution (GPD) estimate for the upper and lower tails. Our approach focuses on the entire distribution rather than the tail distribution only. Finally, a Student’s copula is fitted to the data and used to induce correlation between the simulated residuals of each asset. By using Bayesian MCMC (Markov chain Monte Carlo) to fit the GPD, our estimations do not rely on asymptotics; namely, our estimation gets rid of the influence of limited sample size of extreme events. In order to test the effectiveness of this model we backtest the estimated VaRs over a time period. Empirical results demonstrate that the Bayesian CEVT-Copula based approach outperforms traditional methods such as historical simulation (over-estimate) or conditional normal (underestimate) model.

Keywords: Value-at-Risk (VaR), Conditional Extreme Value Theory (CEVT)

1. Introduction

Value-at-Risk (VaR, Jorion, 2000) is a popular approach to quantifying market risk. It yields an estimate of the likely losses which could rise from price changes over a horizon at a given confidence level. VaR makes risk measure an intuitive criterion for asset management, and hence it very appeals to financial decision makers (Fischer, 2003; Miller, 2003; Rosengarten and Zangari, 2003). Inaccurate portfolio VaR estimates may lead firms to maintain insufficient risk capital reserves so that they have an inadequate capital cushion to absorb large financial shocks. For example, several major financial institutions crashed not long after the breakout of recent financial crises (e.g., East Asian financial crisis of 1997), and some of these failures have been associated with substantial portfolio VaR estimation errors.

Currently, most of the current research on VaR estimation focuses on the univariate case making it undesirable for portfolio risk management. Moreover, most of the significant research contributions to the literature on portfolio VaR are limited to estimators of marginal VaR, component VaR, and incremental VaR instead of portfolio VaR itself (Hallerbach, 2002). This study employs new framework for portfolio VaR estimations, which integrates asymmetric GARCH models for time-varying return distribution of individual assets, extreme value theory (EVT, Embrechts et al., 1997) for tail distributions, and copula functions (Nelsen, 1999) for the dependency structure on all assets of a portfolio. More importantly, we use the Bayesian Markov chain Monte Carlo (MCMC, Robert and Casella, 2004) method to accurately estimate the parameters for extreme value distributions instead of the maximum likelihood estimations (MLE).