Maximal abelian quotient of certain cyclically presented groups arising from manifolds

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Abstract

The fundamental groups of certain 3–manifolds are cyclically presented. We consider the Maximal Abelian quotient, An, of the family of cyclically presented groups

$$G_n = \left\langle x_i : \left(x_{i+1}^{-h}\, x_{i+2}^{h}\right)^{h+1}\, x_{i+1} \left(x_{i+1}^{-h}\, x_i^{h}\right)^{h+1} \right\rangle_n$$

that are isomorphic to the fundamental group of certain Takahashi Manifolds. We show that these Maximal Abelian quotients are 2-generated, isomorphic to $\mathbb{Z}_{d_i} \times \mathbb{Z}_{d2}$, $d_2 \,|\, d_1$ and have orders $\circ(A_n) = 2h^n(h+1)^n(1-\cos n\theta)$, where $\cos\theta = \dfrac{2h(h+1)-1}{2h(h+1)}$. Furthermore, we prove that $\circ(A_n)$ satisfies the recurrence relation $a_1 = 1, a_2 = 4h^2 + 4h - 1, a3 = \left(3h^3 + 3h - 1\right)^2$, $a_n = \left(3h^2 + 3h - 1\right)a_{n-1} - h\left(3h^3 + 6h^2 + 2h - 1\right)a_{n-2} + (h(h+1))^3 a_{n-3}$ for all $n \geq 4$.

Keywords: *Cyclically presented groups, Maximal Abelian quotient, Takahashi Manifolds*