

# An identity derived from the solution of a class of differential equations for the evolution of a key agreement protocol

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## Abstract

In this paper we prove an identity involving the Lambert W function which is derived through the solution of an instance of the Abel differential equation of the first kind. For  $x \in \mathfrak{R}$ , the *Lambert W function* is defined as the principle branch of the function that satisfies the following equation  $\text{Lambert W}(x)e^{\text{Lambert W}(x)} = x$ . The *Abel Differential Equation of the First Kind* is defined as  $\frac{dy(t)}{dt} = f_3(t)y(t)^3 + f_2(t)y(t)^2 + f_1(t)y(t) + f_0(t)$ . The identity is proved through a connection of the solution of a special case of this type of differential equations with the solutions of a third degree polynomial equation.

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