ON LARGE DEVIATIONS FOR SOME DYNAMICAL SYSTEMS AND FOR GIBBS STATES AT ZERO TEMPERATURE

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Abstract. In this article we analyze two issues related with large deviations in dynamical systems:

1.We show that the level-2 large deviation principle established by Comman and Rivera-Letelier[1], is satisfied by maps with a specification property and, with some additional condition, also by those with the *almost property product,* which is weaker than specification. The earlier mentioned authors proved that their principle, which is a generalization of previous results by Kifer, is verified by a class of hyperbolic rational maps.

2. In a previous article[5] we have considered a family of Gibbs states $\{\mu_q\}_{q\geq 1}$, which had an accumulation point μ_∞ (zero temperature limit since the interpretation of q as the inverse of the temperature). We proved that μ_∞ is a maximizing measure for more general systems than symbolics. In a recent article by Lopes and Mengue[4] were considered similar families of states and proved, for symbolic dynamics, that an accumulation point of the family was maximizing. Also they established a large deviation principle. In this note we show how to use the results of our previous work to describe a large deviation process in a more general context than[4].

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1. Introduction

A large deviation process is described in the following way: let $\mathcal{X}$ be a Hausdorff topological vector space and let (ν_n) be a sequence of measures in $\mathcal{X}$, a *level-2 large deviation principle* for the sequence (ν_n) is satisfied when there exists a lower semi-continuous map $I : \mathcal{X} \to [0, +\infty]$ such that

$\lim_{n\to\infty} \sup \dfrac{1}{n} \log \nu_n(F) \leq -\inf \{I(x) : x \in F\}$ if F is a closed subset of $\mathcal{X}$

$\lim_{n\to\infty} \inf \dfrac{1}{n} \log \nu_n(U) \geq -\inf \{I(x) : x \in U\}$ if U is an open subset of.$\mathcal{X}$.

The map I is called the *rate function.*

Let (ν_n) be supported on a compact subset of $\mathcal{X}$ and satisfying a large deviation principle in $\mathcal{X}$ with rate function I, then given any Hausdorff topological space $\mathcal{Y}$ and any map $\phi : \mathcal{X} \to \mathcal{Y}$, a large deviation process in $\mathcal{Y}$ with rate function defined by the functional $y \mapsto \inf \{I(x) : \phi(x) = y\}$ is satisfied for the sequence $(\phi(\nu_n))$. This result is known as the *contraction principle.*

Let us consider the special cases $\mathcal{X} = \widetilde{\mathcal{M}}(X)$, the set of signed measures on a compact set X, $K = \mathcal{M}(X)$ a compact subset, $\mathcal{Y} = \mathbf{R}$ and $\phi(\mu) = \phi_\psi(\mu) = \int \psi d\mu$, for some map $\psi \in C(X, \mathbf{R})$. In this situations the functional is defined as $\alpha \mapsto \inf \left\{I(\mu) : \int \psi d\mu = \alpha\right\}$. Deviation principles obtained from the contraction principle are called *level-1 large deviation principle.*

In [1] a level-2 large deviation principle was formulated as follows: let $f : X \to X$ be a continuous map, such that the entropy map $\mu \mapsto h_\mu(f)$ is upper semi-continuous, let us fix a map $\varphi \in C(X, \mathbf{R})$ and let W be a dense subset of $C(X, \mathbf{R})$, with the property that for any $\psi \in W$ the map $\varphi + \psi$ has an unique equilibrium state. Let (ν_n) be a sequence of Borel measures in $\mathcal{M}(X)$ such that

$\dfrac{1}{n} \log \int_{\mathcal{M}(X)} \exp(n \int \psi d\mu) d\nu_n$ converges, as $n \to \infty$, to $P(\varphi + \psi) - P(\varphi)$, then a level-2 large deviation process is satisfied with rate function