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**ORIGINAL ARTICLE**

# OPTIMIZATION OF MARKOVIAN ARRIVAL AND SERVICE QUEUING MODEL UNDER FUZZINESS

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**Abstract:** In this paper, we discuss the fuzzy optimization of cost of Markovian queuing model with single server. Our aim is to construct the total cost function and further compute the total optimal cost of the system in fuzzy environment in order to provide more realistic results of the model under consideration as compared with crisp results. Finally, the sensitivity analysis has been presented with the help of numerical demonstration of the model.

***Key words*:** Fuzzy optimization, Queuing model, Cost function, Sensitivity analysis.

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## 1. Introduction

Fuzziness implying imprecise and ambiguous information can be handled and modeled for decision making using fuzzy set theory and logic [Belloman and Zadeh (1970), and Zadeh (1978)]. Markovian queues have been a serious concern in the research of queuing theory such as Mishra and Yadav (2010), Mishra and Shukla(2009)**,** Priyaand Sudhesh (2018), Sharma (2016), Singh *et al*. (2016) and Sundari and Palaniammal (2015). It has been observed that in the Markovian queues, fuzzy queuing models are more useful as compared to crisp ones because fuzzy queuing models are more realistic in practical situations. For example, if we talk about mean arrival rate or mean service rate or both it is more possibilistic than probabilistic. At the same time, we also agree that occurrence of arrivals and services are completely probabilistic at service station but their numerical expressions are possibilistic. Apart from this, fuzzy queuing models being much closer to reality, it has wide range of applications as compared to crisp ones; for example, vide, Prado and Fuente (2010).

We can categorize queuing models in two ways. First is known as descriptive and second is known as normative. Descriptive queuing models are actual ones which appear in real situations where as normative are those which should be optimum for the given situations. In this model, we optimize parameters of arrival, service, number of servers, queue discipline and controls *etc.* for the queuing models. Therefore, normative model is an aspirational queuing model where as descriptive one is a real-situation queuing model. Second type of queuing models is known as queuing decision models or design and control models. This category of models attempts to compute the parameters of the model that should optimize them. Amongst control models, service control of queuing models depends on the measures such that service rate, the number of servers, queue discipline, or a combination of factors etc. Some arrival control is also possible by varying arriving customers or assigning arrivals to some servers or controlling arrivals by devising some tolls or other feasible constraints including designing parameter of physical space and working shift etc.

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Of late, the new trend has swung that the queuing model is optimized with uncertain input data. This uncertainty of input data is attempted to figure out the model by using fuzzy paradigm. In fuzzy paradigm, fuzzy optimization methods are applied to optimize functions involved therein containing fuzzy coefficients and parameters related to the models such as Prameela and Kumar (2019) and Palpandi and Geetharamani (2013). Solutions of design and control models of queues in the form of performance measures are obtained by several methods so far designed when cost coefficients, arrival and service parameters are precisely known. But when these parameters and coefficients are imprecise and vary over time (for example, waiting cost per unit may vary over time), the conventional queuing decision models are not sufficient to provide reliable estimate of cost coefficients and queuing related parameters because their imprecision and ambiguity occur due to situations beyond control. In this situation, an intervention is intended to assess the impact as to how the system tends to work. In order to effectively respond on such a situation, fuzzy queuing decision models are needed which can investigate and explore such models in depth [Barak and Fallahnezhad (2012), Chen *et al*. (2020), Fathi-Vajargah and Ghasemalipour (2016), Enrique and Enrique (2014), Kannadasan and Sathiyamoorth (2018), Gou *et al.* (2017), Hidayah *et al.* (2019)].

The paper attempts to discuss both kinds of models. The queuing model in its true situation with one or more parameters as uncertainly known is subjected to optimization which leads to design and control queuing model in fuzzy environment because these are more realistic and practical than its traditional counterpart. Here, we discuss the fuzzy optimization of cost of Markovian queuing model with single server in which fuzzification of parameters by suitable fuzzy set, evaluation of certain fuzzy operations and defuzzification by signed distance method are made and thereafter the task of optimization and computing is carried out. Our aim is to construct the total cost function in fuzzy environment and subject it under optimization. This mathematical intervention provides us a nonlinear equation which is solved by R 3.6.2 for its optimum performance measure in the form of optimal total cost of the queuing system under consideration. Finally, the sensitivity analysis has been presented with the help of numerical demonstration of the model. This model is presumably believed to ensure cost-effective and efficient system of manpower-workflow in order to smoothen the traffic flow in the organizations.

## 2. Notations and Assumptions

The notations and assumptions used in this paper are as follows:

### 2.1 Notations

$TC$ = Optimal Total Cost (OTC),

$k$ = Service cost per unit (SC),

$c$ = Waiting cost per unit (WC)

$\lambda$ = Arrival rate of customer (ARC),

$\mu$ = Service rate (SR),

$\mu^*$ = Optimal Service Rate (OSR),

$\tilde{k}$ = Fuzzified Service cost per unit (FSC),

$\tilde{c}$ = Fuzzified Waiting cost per unit(FWC)

$\tilde{\lambda}$ = Fuzzified Arrival Rate (FAR),

$\tilde{TC}$ = Fuzzified Optimal Total Cost (FOTC)

### 2.2 Assumptions

Queuing model assumes that customers' arrival follow Poisson distribution and service times the exponential distribution.

## 3. Model Development and Analysis

Here, we consider Markovian queuing model whose arrival and service both follows Poisson probability law and it has single server with first come and first served discipline as well as infinite capacity. There are several operating characteristics or performance measures of the queuing model namely expected number of customers in queue and system expected waiting time in queue and system; and service utilization factor or busy period. In most of papers, these operation characteristics are aimed to discuss but research on optimal total cost of the queuing system is a dearth more particularly in fuzzy environment.

Cost model of queuing system is developed on two important costs- service cost and waiting cost which conflict each other. This implies that a decrease in the level of service would increase the cost of waiting. The cost model can be expressed as $TC = k\,\mu + cE(n)$. This implies that $TC = k\,\mu + \dfrac{c\lambda}{\mu - \lambda}$,

where, $k$ = cost of service rate per unit time, $c$ = cost of waiting customers per unit time, and $E(n)$ = average number of customers in the system.

### 3.1 Fuzzy mathematical formulation

Total model cost function is defined as $TC = k\,\mu + \dfrac{c\lambda}{\mu-\lambda}$.

For minimum cost w.r.t. service, we have

$$\frac{d}{d\mu}[TC] = \frac{d}{d\mu}\left[k\,\mu + \frac{c\lambda}{\mu-\lambda}\right] = k - \frac{c\lambda}{(\mu-\lambda)^2} = 0$$

For minimum cost, we must have

$$\frac{d^2}{d\,\mu}[TC] = \frac{2c\lambda}{(\mu-\lambda)^3} > 0$$

Further, we define a trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ with membership function $\mu_A(x)$ as

$$\mu_A(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, when\ a \le x \le b \\ R(x) = \dfrac{d-x}{d-c}, when\, c \le x \le d \\ 0 \qquad\qquad\quad , otherwise \end{cases}$$

Now, we wish to fuzzify cost coefficients and arrival rates $k$, $c$, $\lambda$ with the help of trapezoidal fuzzy numbers as $\tilde{k}, \tilde{c}$ and $\tilde{\lambda}$ respectively.

$\tilde{k} = (k_1, k_2, k_3, k_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$ $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$

$$\tilde{T}C = \tilde{k}\,\mu + \frac{\tilde{c}\,\tilde{\lambda}}{\mu - \tilde{\lambda}}$$ which implies that

$$\tilde{T}C = (k_1\mu, k_2\mu, k_3\mu, k_4\mu)$$

$$+\left(\frac{c_1\lambda_1}{-\lambda_1}, \frac{c_2\lambda 2}{-\lambda_2}, \frac{c_3\lambda_3}{-\lambda 3}, \frac{c_4\lambda_4}{\mu-\lambda_4}\right)$$

which finally turns out to be as

$$= \left(k_1\mu - c_1,\ k_2\mu - c_2,\ k_3\mu - c_3, k_4\mu - \frac{c_4\lambda_4}{\mu-\lambda_4}\right)$$

$$\tilde{T}C = (W, X, Y, Z)$$

where, $W = k_1\mu - c_1, X = k_2\mu - c_2,$

$$Y = k_3\mu - c_3, Z = k_4\mu - \frac{c_4\lambda_4}{\mu-\lambda_4}$$

Now we define

$$c_2(\alpha) = W + (X - W)\alpha$$

$$= (k_1\mu - c_1)\left[\left(k_2\mu - c_2 - (k_1\mu - c_1)\right)\right]\alpha$$

$$= (k_1\mu - c_1) + \left[(k_2 - k_1)\mu + (c_1 - c_2)\right]\alpha$$

and

$$CR(\alpha) = Z - (Z - Y)\alpha = \left(k_4\mu + \frac{c_4\lambda_4}{\mu-\lambda_4}\right)$$

$$-\left(k_4\mu + \frac{c_4\lambda_4}{\mu-\lambda_4} - k_3\mu + c_3\right)\alpha$$

$$= \left(k_4\mu + \frac{c_4\lambda_4}{\mu-\lambda_4}\right) - \left(k_4\mu + \frac{c_4\lambda_4}{\mu-\lambda_4} - k_3\mu + c_3\right)\alpha$$

Next, we define

$$\breve{T}C_{ds} = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_R(\alpha)]\,\alpha\, d\alpha,$$

$$\tilde{T}C_{ds} = \frac{1}{2}\left[(k_1\mu - c_1) + \left(k_4\mu + \frac{c_4\lambda_4}{\mu-\lambda_4}\right)\right]$$

$$+\frac{1}{4}\left[(k_2 - k_1 + k_3 - k_4)\mu + (c_1 - c_2 - c_3) - \frac{c_4\lambda_4}{\mu-\lambda_4}\right]$$

$$\tilde{T}C_{ds} = \frac{1}{4}\left[(k_1 + k_2 + k_3 + k_4)\mu - (c_1 + c_2 + c_3) + \frac{c_4\lambda_4}{\mu-\lambda_4}\right]$$

Now, we differentiate the above expression w.r.t. μ as

$$\frac{d}{d\mu}[TC_{ds}] = \frac{1}{4}\left[(k_1 + k_2 + k_3 + k_4) - \frac{c_4\lambda_4}{(\mu-\lambda_4)^2}\right]$$

For minimum TC, $\dfrac{d}{d\mu}[TC_{ds}] = 0$ with sufficient condition $\dfrac{d^2}{d\mu^2}[TC_{ds}] = \dfrac{1}{2}\dfrac{c_4\lambda_4}{(\mu-\lambda_4)^3} > 0$ which ultimately gives us $(k_1 + k_2 + k_3 + k_4)$ $(\mu-\lambda_4)^2 - c_4\lambda_4 = 0$ .

This nonlinear equation is solved by R using the following computational flowchart for optimum service rate $\mu^*$, which in turn provides us total optimal cost. The optimal results are provided in Tables 4-6 which are easily comparable with the results obtained for crisp model given in Tables 1-3.

## 4. Computing Flowchart

The following flowchart is computed for obtaining the optimal service rate and total optimal cost of the model.

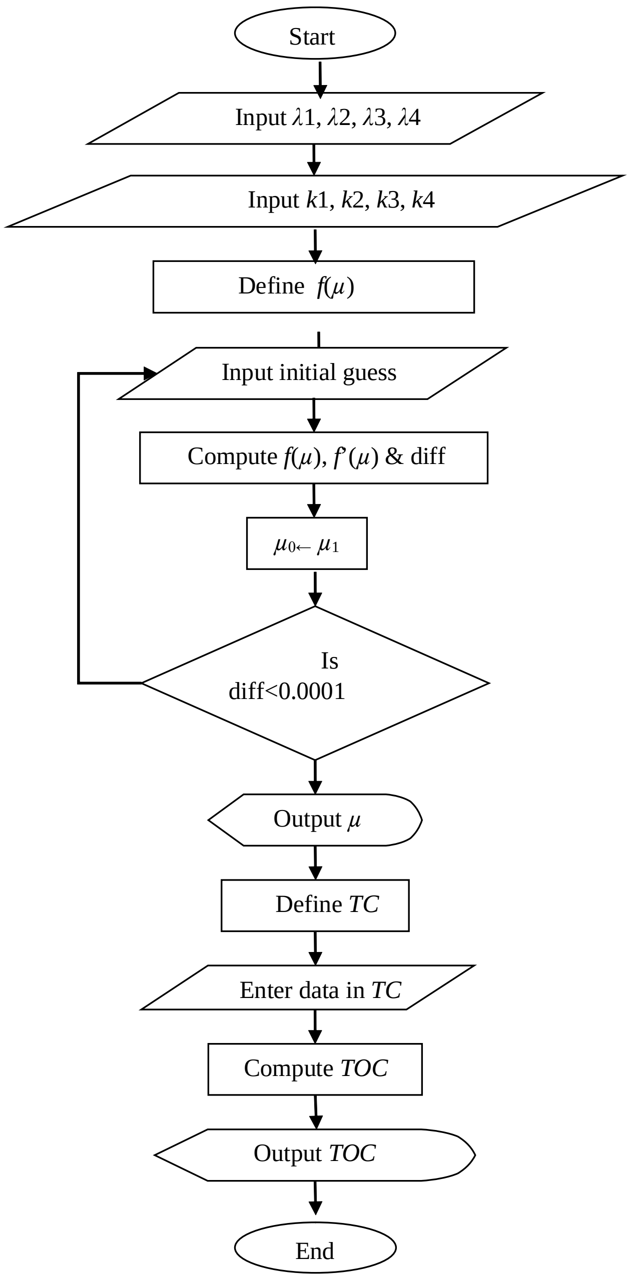


**Table 1:** Computation table for *k*, *TC*.

| | *k* | *c* | $\lambda$ | $\mu$ | *TC* |
|---|---|---|---|---|---|
| **Case 1** | 13 | 9 | 5 | 6.86 | 113.37 |
| | 15 | 9 | 5 | 6.73 | 126.96 |
| | 17 | 9 | 5 | 6.63 | 140.32 |
| | 19 | 9 | 5 | 6.54 | 153.48 |

**Table 2:** Computation table for *C*, *TC*.

| | *c* | *k* | $\lambda$ | $\mu$ | *TC* |
|---|---|---|---|---|---|
| **Case 2** | 9 | 13 | 5 | 6.86 | 113.37 |
| | 11 | 13 | 5 | 7.06 | 118.48 |
| | 13 | 13 | 5 | 7.24 | 123.14 |
| | 15 | 13 | 5 | 7.40 | 127.45 |

**Table 3:** Computation table for $\lambda$ , *TC*.

| | $\lambda$ | *c* | *k* | $\mu$ | *TC* |
|---|---|---|---|---|---|
| **Case 3** | 5 | 9 | 13 | 6.86 | 113.37 |
| | 7 | 9 | 13 | 9.20 | 148.24 |
| | 9 | 9 | 13 | 11.50 | 163.46 |
| | 11 | 9 | 13 | 13.76 | 214.75 |

## 5. Results and Sensitivity Analysis

### 5.1 Results obtained for crisp model

The results obtained for the crisp model are presented in Tables 1-3 respectively.

### 5.2 Results obtained for fuzzy model

The results obtained for the fuzzy model are presented in Tables 4-6.

### 5.3 Sensitivity analysis

A sensitivity analysis is altogether a study of variation-propensity of the model. For the crisp model, Table 1 shows that the total optimal cost of the model increases with the increase in service cost per unit. In Table 2, it is worth notable that whenever per unit waiting cost, per unit time increases, total optimal cost of the model also increases. Similarly, arrival rate of the customer to the service channel whenever increases, total optimal cost of the model also increases, which is depicted by the Table 3 as well as Fig. 1.

For the fuzzy model, Table 4 show that the increase in fuzzy service cost per unit increases in total optimal fuzzy cost of the model under consideration. Table 5 represents that the fuzzy waiting cost per unit if

**Table 4:** Computation table for $\tilde{k}$ $\tilde{T}C$.

| | $\tilde{k}$ | | | | $\tilde{c}$ | | | | $\tilde{\lambda}$ | | | | $\mu$ | $\tilde{T}C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\mu$ | $\tilde{T}C$ |
| **Case 1** | 10 | 12 | 14 | 16 | 6 | 8 | 10 | 12 | 2 | 4 | 6 | 8 | 9.36 | 133.33 |
| | 12 | 14 | 16 | 18 | 6 | 8 | 10 | 12 | 2 | 4 | 6 | 8 | 9.26 | 151.95 |
| | 14 | 16 | 18 | 20 | 6 | 8 | 10 | 12 | 2 | 4 | 6 | 8 | 9.19 | 170.40 |
| | 16 | 18 | 20 | 22 | 6 | 8 | 10 | 12 | 2 | 4 | 6 | 8 | 9.12 | 188.71 |

**Table 5:** Computation table for $\tilde{c}$ $\tilde{T}C$.

| | $\tilde{c}$ | | | | $\tilde{k}$ | | | | $\tilde{\lambda}$ | | | | $\mu^*$ | $\tilde{T}C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\mu$ | $\tilde{T}C$ |
| **Case 2** | 6 | 8 | 10 | 12 | 10 | 12 | 14 | 16 | 2 | 4 | 6 | 8 | 9.36 | 133.33 |
| | 8 | 10 | 12 | 14 | 10 | 12 | 14 | 16 | 2 | 4 | 6 | 8 | 9.47 | 134.66 |
| | 10 | 12 | 14 | 16 | 10 | 12 | 14 | 16 | 2 | 4 | 6 | 8 | 9.57 | 135.79 |
| | 12 | 14 | 16 | 18 | 10 | 12 | 14 | 16 | 2 | 4 | 6 | 8 | 9.66 | 136.77 |

**Table 6:** Computation table for $\tilde{\lambda}$ $\tilde{T}C$.

| | $\tilde{\lambda}$ | | | | $\tilde{c}$ | | | | $\tilde{k}$ | | | | $\mu^*$ | $\tilde{T}C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $\mu$ | $\tilde{T}C$ |
| **Case 3** | 2 | 4 | 6 | 8 | 6 | 8 | 10 | 12 | 10 | 12 | 14 | 16 | 9.36 | 133.33 |
| | 4 | 6 | 8 | 10 | 6 | 8 | 10 | 12 | 10 | 12 | 14 | 16 | 11.52 | 134.66 |
| | 6 | 8 | 10 | 12 | 6 | 8 | 10 | 12 | 10 | 12 | 14 | 16 | 13.66 | 193.27 |
| | 8 | 10 | 12 | 14 | 6 | 8 | 10 | 12 | 10 | 12 | 14 | 16 | 15.80 | 222.73 |

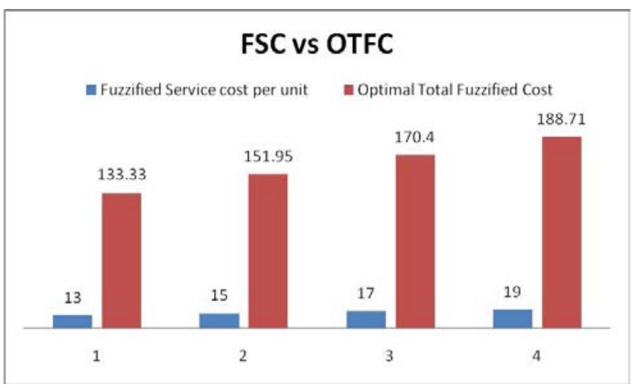


**Fig. 1:** Variation of optimal total fuzzified cost and fuzzified service cost/unit

increases, then total optimal fuzzy cost of the model also increases. At last in Table 6, it may be observed that the fuzzy arrival rate of customers to the service channel whenever increases, it results increase in total optimal fuzzy cost of the model.

Correlation is a vital criterion to judge the validity of the model for its application. Thus, in both of the cases of crisp and fuzzy, correlation between service, waiting and arrival rate as one variable and total optimal cost of the model as another variable are positive. The only basic difference between two environments lies in the fact that the correlation in former case is lesser positive as compared to later case.

## 6. Conclusion

In the present investigation, we developed a queuing model in fuzzy environment which is more close to reality when measurements are beyond our control. The computing of total optimal cost of the queuing model with single server in fuzzy environment is an improved solution to the existing queuing model. The results obtained in Section 5 are easily comparable in both of the environments the crisp and the fuzzy. The extension of such queuing model to the fuzzy environment can provide us broader application with elements of uncertainty. For researchers, this paper is presumably believed to provide more information and robust results in order to design Markovian queuing models for fuzzy environment. Such Markovian models are very useful to develop more efficient waiting line system to deal with staffing needs and settlement, pricing, management of arrivals, service quality, reduction of customer' waiting time and increasing serviced customers etc. These models can also serve to provide queuing performance measures as operational management strategies to determine scheduling and inventory control to improve customer-service in the organizations where queues are obviously formed. This technique of Markovian queuing model is also useful for six sigma practitioners to improve the customer-service in the organizations. For future research, fuzzy neuro and intuitionistic fuzzy approaches will be more realistic to study such queuing models.

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