Int. J. Agricult. Stat. Sci. Vol. 14, Supplement 1, pp. 177-185, 2018 ***ISSN : 0973-1903***

ORIGINAL ARTICLE

THE TECHNIQUE OF MULTISTAGE PARTITIONED RANKED SET SAMPLING FOR ESTIMATION OF THE POPULATION MEAN

K. B. Panda and M. Samantaray*

Department of Statistics, Utkal University, Bhubaneswar - 751 004, India.
E-mail : so.sweet.madhusmita@gmail.com

Abstract : In this paper, we propose a new technique for estimating the population mean called as Multistage Partitioned Ranked Set sampling (MPRSS) using the notion of Partitioned Ranked set Sampling (PRSS). It may be served as a generalisation of PRSS. The estimator formed under the newly suggested technique is found to be more efficient than its competing estimators. An empirical study has been carried out to support the theoretical findings.

Key words : Partitioned Ranked Set Sampling, Ranked Set Sampling, Simple Random Sampling, Population Mean, Efficiency.

1. Introduction

McIntyre (1952) suggested a new technique called Ranked Set Sampling (RSS) for estimating the population mean. Dell and Clutter (1972) showed that mean of RSS is unbiased estimator of the population mean, even at errors in ranking. MRSS technique was introduced by Muttalak (1997). Al-Saleh and Al-Omari (2002) considered Multistage Ranked Set Sampling which increased the efficiency of estimating the population mean. Jemain *et al.* (2007) suggested Multistage Median Ranked Set Sampling (MMRSS), whereas Panda and Samantaray (2017) proposed a new technique for estimation of partitioned and double partitioned ranked set sampling.

In this paper, multistage partitioned sampling method is considered for estimation of population mean for different symmetric and asymmetric distributions. This paper consists of the following sections: In Section 2, procedure for sampling method related to this paper are summarized in detail. In Section 3, population mean for MPRSS is given. In Section 4, a simulation study is undertaken. Error sampling in ranking using this technique is presented in Section 5. Finally in Section 6, conclusions are given.

2. Sampling Methods

Ranked Set Sampling (RSS)

In RSS, divide m^2 units randomly into m sets, each of size m. Without knowing any values for the variable of interest, rank the units within each set with respect to variable of interest based on personal and/or professional judgement or based on a concomitant variable correlated with the variable of interest. Select the smallest ranked unit in the first set, the second smallest ranked unit in the second set and continue the process till the largest ranked unit is selected from the last set. If m is small, then the cycle may be repeated r times so as to obtain a combined sample of size mr.

Median Ranked Set Sampling (MRSS)

MRSS method is carried out by selecting m random samples each of size m units from the population of interest and ranking of the units is done in each sample with respect to a variable of interest. If the sample size m is even, select the smallest ranked units from first m/2 samples, and highest ranked units from last m/2 samples. If the sample size m is odd, select smallest ranked unit from first (m-1)/2 samples, the highest ranked unit from last (m-1)/2 samples and median from middle sample. The cycle can be repeated r times to get a sample of size mr units.

Partitioned Ranked Set Sampling (PRSS)

The PRSS procedure as proposed by Panda and Samantaray (2017) depends on selecting m random samples of size m units from the population and the ranking of units is carried out within each sample with

****Author for correspondence*** ***Received*** *May 18, 2018* ***Revised*** *September 08, 2018* ***Accepted*** *September 22, 2018*

respect to a variable of interest. If the sample size is odd, then select p (m + 1)th rank from first m(m-1)/2 sample and q (m+1)th rank from last m(m-1)/2 samples with median from middle set. If the sample size is even, then select p(m + 1)th rank from first m/2 sets and select q(m+1)th rank from last m/2 sets. The cycle can be repeated r times to get a sample size of mr units. Note that we always take the nearest integer of p(m+1)th and q(m+1)th observations, where p + q = 1.

Double Partitioned Ranked Set Sampling (DPRSS)

DPRSS technique comprises the following steps:

Select m^3 elements from the target population and divide these elements randomly into m^2 sets each of size m.

a. If sample size m is even, select the [p(m + 1)]th rank from each set out of first $m^2/2$ sets and from the second $m^2/2$ sets the [q(m + 1)]th rank from each set is selected.

b If sample size m is odd, select from the first (m(m-1)/2) sets, the (p(m + 1))th rank from each set, the median from next m sets and from last (m(m-1)/2) sets, select (q(m + 1))th rank from each set.

Here, p & q stand for *p*th and *q*th partitioned observations, such that p + q = 1, for example, p = 25% and q = 75% of the observations given. This can be done after arranging the series either in ascending or in descending order visually.

Applying PRSS procedure on m sets obtained in above, we arrive at DPRSS procedure for sample of size m. The whole cycle may be repeated r times to obtain a sample size of mr from DPRSS. From above, we have to examine mr samples out of m^3r population size using DPRSS.

Here, we have to remember that the ranking should be done by visual inspection or by any economical procedure and actual quantification is done at final stage.

Multistage Ranked Set Sampling (MSRSS)

According to multistage ranked set sampling procedure, we have to identify m^{s+1} elements from the target population and divide these elements into m^s sets, each of size m units. Then apply RSS produce for s times to have final set containing m units. Again the whole cycle can be repeated r times to get a sample of size mr units.

Multistage Median Ranked Set Sampling (MMRSS)

The MMRSS method is carried out by selecting m^s random samples each of size m from target population and ranking of the units in each sample with respect to a variable of interest. If the sample size m is even, select the lowest rank from first $\frac{m^s}{2}$ sets and the highest rank from last $\frac{m^s}{2}$ sets. If the sample size m is odd, select the lowest rank from first $\frac{m^{s-1}(m-1)}{2}$ sets and the highest rank from last $\frac{m^{s-1}(m-1)}{2}$ sets with median from middle sets. The cycle can be repeated r times to have sample of size mr units.

Multistage Partitioned Ranked Set Sampling (MPRSS)

The procedure of MPRSS is described in the following steps:

Step 1: Select m^{s+1} units from the population randomly and allocate them into m^s sets each of size m, where s is number of stages and m is the sample size.

Step 2 : From each set of m^s, if the number of sample size is even, select the (p(m+1))th rank from first $(m^s/2)$ sets and the (q(m+1))th rank from last $(m^s/2)$ sets. Similarly , if the sample size is odd, select (p(m+1))th rank from first $(m^{s-1}(m-1)/2)$ sets, from next m^{s-1} sets select the median and from last $((m^{s-1})(m-1)/2)$ sets select (q(m+1))th rank. These steps give size of m^s units out out of m^{s+1} units which will be the sample for next stage of MPRSS.

Step 3 : Repeat the Step 2 on the m^{s-1} partitioned ranked sets each of size m to obtain m^{s-2} samples in second stage.

Step 4 : The process will continue till we have m samples at *s*th stage of partitioned ranked set.

The whole process can be repeated r times, if needed to have size of mr from MPRSS data. If s = 1, MPRSS procedure will give sample of PRSS. Similarly if s = 2, this procedure will give sample of DPRSS. All the stages are done visually or by any cheap method and actual quantification is only done on last sample of size m that is obtained at final stage.

To clarify the procedure, following examples are considered both for odd and even number of sample size.

Example 2.1 : (if the number of sample size is odd)

Let, m =3 and s = 3, so that we have a random sample of size m^{s+1} = 8. Allocate them into 27 sets each of size 3.

Again, $Y_{j(i;m)}^{(s)}$ = *i*th minimum (i = 1, 2, 3) of *j*th set (j=1,2,...,27) at stage 3. After ranking, the units within each set can be written as below:

$$U_1^{(0)} = \left[X_{1(1;3)}^{(0)}, X_{1(2;3)}^{(0)}, X_{1(3;3)}^{(0)}\right],$$

$$U_2^{(0)} = \left[X_{2(1;3)}^{(0)}, X_{2(2;3)}^{(0)}, X_{2(3;3)}^{(0)}\right], ...,$$

$$U_{27}^{(0)} = \left[X_{27(1;3)}^{(0)}, X_{27(2;3)}^{(0)}, X_{27(3;3)}^{(0)}\right]$$

Now, by applying MPRSS procedure on each of the 27 sets, the first (p(m+1))th partitioned value is the smallest rank and (q(m+1))th rank is largest rank. Thus for s = 1, we will select (p(m+1))th rank from (m^{s-1} (m-1)/2) = 9 sets, from next (m^{s-1}) = 9 sets select median and select (q(m+1))th rank from last (m^{s-1}(m-1)/2) = 9 sets.

The MPRSO sample is so obtained as

$$X_{1(1;3)}^{(1)} = min\left(U_1^{(0)}\right),\ X_{2(1;3)}^{(1)} = min\left(U_2^{(0)}\right), ...,$$

$$X_{9(1;3)}^{(1)} = min\left(U_9^{(0)}\right),\ X_{10(M;3)}^{(1)} = med\left(U_{10}^{(0)}\right), ...,$$

$$X_{18(M;3)}^{(1)} = med\left(U_{18}^{(0)}\right),\ X_{19(3;3)}^{(1)} = max\left(U_{19}^{(0)}\right), ...,$$

$$X_{27(3;3)}^{(1)} = max\left(U_{27}^{(0)}\right)$$

The above step gives 9 sets each of the size 3 at the first stage. Hence, the new sets are

$$U_1^{(1)} = \left[X_{1(1;3)}^{(1)}, X_{2(1;3)}^{(1)}, X_{3(1;3)}^{(1)}\right], ...,$$

$$U_3^{(1)} = \left[X_{7(1;3)}^{(1)}, X_{8(1;3)}^{(1)}, X_{9(1;3)}^{(1)}\right],$$

$$U_4^{(1)} = \left[X_{10(M;3)}^{(1)}, X_{11(M;3)}^{(1)}, X_{12(M;3)}^{(1)}\right], ...,$$

$$U_6^{(1)} = \left[X_{16(M;3)}^{(1)}, X_{17(M;3)}^{(1)}, X_{18(M;3)}^{(1)}\right],$$

$$U_7^{(1)} = \left[X_{19(3;3)}^{(1)}, X_{20(3;3)}^{(1)}, X_{21(3;3)}^{(1)}\right], ...,$$

$$U_9^{(1)} = \left[X_{25(3;3)}^{(1)}, X_{26(3;3)}^{(1)}, X_{27(3;3)}^{(1)}\right]$$

For s = 2, apply the DPRSSO methods on these 9 sets, we will have (p(m+1))th rank from 3 sets, select median from next 3 sets and select (q(m+1))th rank from last 3 set. The MPRSSO samples are obtained as

$$X_{1(1;3)}^{(2)} = min\left(U_1^{(1)}\right), X_{2(1;3)}^{(2)} = min\left(U_2^{(1)}\right),$$

$$X_{3(1;3)}^{(2)} = min\left(U_3^{(1)}\right), X_{4(M;3)}^{(2)} = med\left(U_4^{(1)}\right),$$

$$X_{5(M;3)}^{(2)} = med\left(U_5^{(1)}\right), X_{6(M;3)}^{(2)} = med\left(U_6^{(1)}\right),$$

$$X_{7(3;3)}^{(2)} = max\left(U_7^{(1)}\right), ..., X_{9(3;3)}^{(2)} = max\left(U_9^{(1)}\right)$$

The above sample can be rearranged as

$$U_1^{(2)} = \left[X_{1(1;3)}^{(2)}, X_{2(1;3)}^{(2)}, X_{3(1;3)}^{(2)}\right],$$

$$U_2^{(2)} = \left[X_{4(M;3)}^{(2)}, X_{5(M;3)}^{(2)}, X_{6(M;3)}^{(2)}\right],$$

$$U_3^{(2)} = \left[X_{7(3;3)}^{(2)}, X_{8(3;3)}^{(2)}, X_{9(3;3)}^{(2)}\right]$$

Again for s = 3, applying the PRSSO method on above 3 sets, we have

$$X_{1(1;3)}^{(3)} = min\left(U_1^{(2)}\right), X_{2(M;3)}^{(3)} = med\left(U_2^{(2)}\right),$$

$$X_{3(3;3)}^{(3)} = max\left(U_3^{(2)}\right).$$

The final set $\left[X_{1(1;3)}^{(3)}, X_{2(M;3)}^{(3)}, X_{3(3;3)}^{(3)}\right]$ is the required Third Stage Ranked Set Sample. It is of interest to note that the samples are i.i.d. random variable. These 3 units are exactly measured.

Example 2.2: (if the number of sample size is even)

For even sample size, we have to apply MPRSSE method, which may be described as follows

Let m = 4, s = 3, then we have to select random sample of 64 sets, each should contain 4 units. $Y_{j(i;m)}^{(s)} = i$th minimum (i = 1, 2, 3, 4) of set (j = 1, 2, ..., 64) at stage s.

After ranking, the unit within each subset will be

$$U_1^{(0)} = \left[X_{1(1;4)}^{(0)}, X_{1(2;4)}^{(0)}, X_{1(3;4)}^{(0)}, X_{1(4;4)}^{(0)}\right],$$

$$U_2^{(0)} = \left[X_{2(1;4)}^{(0)}, X_{2(2;4)}^{(0)}, X_{2(3;4)}^{(0)}, X_{2(4;4)}^{(0)}\right], ...,$$

$$U_{64}^{(0)} = \left[X_{64(1;4)}^{(0)}, X_{64(2;4)}^{(0)}, X_{64(3;4)}^{(0)}, X_{64(4;4)}^{(0)}\right]$$

Now, applying MPRSSE method on each 64 sets. The first Partitioned value (p(m+1))th (for p = 10%) = 10%(4+1)=0. 5th observation, which indicates the first or lowest observation from each of first $\frac{m^s}{2}=32$ sets and last partitioned value (q(m+1))th (for q= 90%)= 90%(4+1) = 4. 5th observation indicate fourth observation or highest observation from last $\frac{m^s}{2}=32$ sets.

Using the above Procedure, we arrive at

$$X_{1(1;4)}^{(1)}=min\left(U_1^{(2)}\right), X_{2(1;4)}^{(1)}=min\left(U_2^{(0)}\right),...,$$

$$X_{32(1;4)}^{(1)}=min\left(U_{32}^{(0)}\right), X_{33(4;4)}^{(1)}=max\left(U_{33}^{(0)}\right), ...,$$

$$X_{64(4;4)}^{(1)}=max\left(U_{64}^{(0)}\right)$$

The above observations can be reorganised in the following 16 sets

$$U_1^{(1)}=\left[X_{1(1;4)}^{(1)}, X_{2(1;4)}^{(1)}, X_{3(1;4)}^{(1)}, X_{4(1;4)}^{(1)}\right],....,$$

$$U_8^{(1)}=\left[X_{29(1;4)}^{(1)}, X_{30(1;4)}^{(1)}, X_{3(1;4)}^{(1)}, X_{32(1;4)}^{(1)}\right],$$

$$U_9^{(1)}=\left[X_{33(4;4)}^{(1)}, X_{34(4;4)}^{(1)}, X_{35(4;4)}^{(1)}, X_{36(4;4)}^{(1)}\right],....,$$

$$U_{16}^{(1)}=\left[X_{6(4;4)}^{(1)}, X_{62(4;4)}^{(1)}, X_{63(4;4)}^{(1)}, X_{64(4;4)}^{(1)}\right]$$

Now applying DPRSSE procedure in above sets, we will get finalised sample set as $\left[X_{1(1;4)}^{(3)}, X_{2(1;4)}^{(3)}, X_{3(4;4)}^{(3)}, X_{4(4;4)}^{(3)}\right]$ are i.i.d. random variables. These 4 observation are to be actually measured.

3. Estimation of Population Mean

Let X_1, X_2, ..., X_m be a random sample with pdf f(x) and cdf F(x) with mean μ and variance σ^2. Again let X_{11}, X_{12}, ..., X_{1m}; X_{21}, X_{22}, ..., X_{2m};...; X_{m1}, X_{m2}, ..., X_{mm}, be independent random variables.

The SRS estimator of the population mean from a sample of size m is given by

$$\overline{X}_{SRS}=\frac{1}{m}\sum_{i=1}^{m}X_i$$

and variance $Var\left(\overline{X}_{SRS}\right)=\frac{\sigma^2}{m}$ (1)

The RSS estimator of the population mean from a sample of size m [McIntyre (1952)] is given by

$$\overline{X}_{RSS}=\frac{1}{m}\sum_{i=1}^{m}X_{(i;m)}$$

and variance $Var\left(\overline{X}_{RSS}\right)=\frac{\sigma^2}{m}-\frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{(i;m)}-\mu\right)^2$ (2)

where, $\sum_{i=1}^{m}\left(\mu_{(i;m)}-\mu\right)^2>0$

So, $\overline{X}_{RSS}$ is more efficient than $\overline{X}_{SRS}$, based on same number of observations.

The DPRSS estimator of the population mean from a sample size of m [Panda and Samantaray (2017)] is given by

$$\overline{X}_{DPRSSE}=\frac{1}{m}\left[\sum_{i=1}^{l}X_{i(p(m+1);m)}^{(2)}+\sum_{i=l+1}^{m}X_{i(q(m+1);m)}^{(2)}\right] \quad (3)$$

$$\overline{X}_{DPRSSO}=\frac{1}{m}\left[\sum_{i=1}^{h}X_{i(p(m+1);m)}^{(2)}+\sum_{i=h+2}^{m}X_{i(q(m+1);m)}^{(2)} + X_{h+1(M;m)}^{(2)}\right] \quad (4)$$

and variance is given by

$$Var\left(\overline{X}_{DPRSSE}\right)=\frac{1}{m^2}\left[\sum_{i=1}^{h}Var\left(X_{i(p(m+1);m)}^{(2)}\right)+\sum_{i=l+1}^{m}Var\left(X_{i(q(m+1);m)}^{(2)}\right)\right] \quad (5)$$

$$Var\left(\overline{X}_{DPRSSO}\right)=\frac{1}{m^2}\left[\sum_{i=1}^{h}Var\left(X_{i(p(m+1);m)}^{(2)}\right)+\sum_{i=h+2}^{m}Var\left(X_{i(q(m+1);m)}^{(2)}\right)+Var\left(X_{h+1(M;m)}^{(2)}\right)\right] \quad (6)$$

Hence, $\overline{X}_{DPRSS}$ is more efficient than $\overline{X}_{SRS}$ as well as $\overline{X}_{SRS}$, based on same number of observations.

It is of interest to note that, the DPRSS method constitute by applying the usual RSS method in partitioned sample on m^2 sets each of size m up to second stage, which is different from our work based on MPRSS, where we apply PRSS method on m^s sets each of size m up to *s*th stage.

To estimate the population mean using MSPRSS method, at the sth stage for odd number of sample size, Let $X_{i(p(m+1);m)}^{(s)}$ be the (p(m+1))h rank from *i*th sets (i = 1, 2, ..., h) and $X_{i(q(m+1);m)}^{(s)}$ be the (q(m+1))th ranks from *i*th sets (i=h+2, ..., m) with median from h=((m-1)/2) set. Here, $X^{(s)}_{1(p(m+1);m)}, X^{(s)}_{2(p(m+1);m)}, ..., X^{(s)}_{h(p(m+1);m)}$, $X^{(s)}_{h+1(M;m)}$, $X^{(s)}_{h+2(q(m+1);m)}$, ..., $X^{(s)}_{m(q(m+1);m)}$ are i.i.d. random variable. All units are mutually independent but not identically distributed. These measured units denotes sample of MPRSSO. Similarly, for even number of sample, let $X^{(s)}_{i(p(m+1);m)}$ be the (p(m+1))th rank from *i*th sets (i = 1, 2, ..., l) and $X^{(s)}_{i(q(m+1);m)}$ be the (q(m1))th rank from last *i*th sets (i = l+1, ..., m). Hence, MPRSSE contain $X^{(s)}_{1(p(m+1);m)}, X^{(s)}_{2(p(m+1);m)}, \ldots, X^{(s)}_{m/2(p(m+1);m)}$, $X^{(s)}_{((m/2)+1)(q(m+1);m)}, \ldots, X^{(s)}_{m(q(m+1);m)}$ are i.i.d. and mutually independent units, but not identically distributed.

The MSPRSS estimators of population mean in case of an even and odd sample sizes respectively are given by

$$\overline{X}_{MPRSSE} = \frac{1}{m}\left[\sum_{i=1}^{l} X^{(s)}_{i(p(m+1);m)} + \sum_{i=l+1}^{m} X^{(s)}_{i(q(m+1);m)}\right],$$

where, l = m/2 (7)

$$\overline{X}_{MPRSSO} = \frac{1}{m}\left[\sum_{i=1}^{h} X^{(s)}_{i(p(m+1);m)} + \sum_{i=l+2}^{m} X^{(s)}_{i(q(m+1);m)} + X^{(s)}_{h+1(M;m)}\right] \quad (8)$$

where, h = ((m-1)/2)

and variance is given by

$$Var\left(\overline{X}_{MPRSSE}\right) = \frac{1}{m^2}\left[\sum_{i=1}^{h} Var\left(X^{(s)}_{i(p(m+1);m)}\right) + \sum_{i=l+1}^{m} Var\left(X^{(s)}_{i(q(m+1);m)}\right)\right]$$

$$= \frac{1}{2m}\left[Var\left(X^{(s)}_{i(p(m+1);m)}\right) + Var\left(X^{(s)}_{i(q(m+1);m)}\right)\right] \quad (9)$$

$$Var\left(\overline{X}_{MPRSSO}\right) = \frac{1}{m^2}\left[\sum_{i=1}^{h} Var\left(X^{(s)}_{i(p(m+1);m)}\right) + \sum_{i=h+2}^{m} Var\left(X^{(s)}_{i(q(m+1);m)}\right) + Var\left(X^{(s)}_{h+1(M;m)}\right)\right]$$

$$= \frac{m-1}{2m^2}\left[Var\left(X^{(s)}_{i(p(m+1);m)}\right) + Var\left(X^{(s)}_{i(q(m+1);m)}\right)\right] + \frac{1}{m^2} Var\left(X^{(s)}_{h+1(M;m)}\right) \quad (10)$$

The properties of the MPRSS estimators are

(1) If the distribution is symmetric about the population mean, then

a. The MPRSS estimator is unbiased estimator of population mean.
b. The efficiency of is MPRSS estimator is increasing with increase of number of stage s.

$$Var\left(\overline{X}^{(s)}_{MSPRSS}\right) < Var\left(\overline{X}^{(s)}_{RSS}\right), \text{ for } s > 2.$$

(2) If the distribution is asymmetric about the population mean, then the efficiency as well as MSE gives better result than RSS, as

$$Var\left(\overline{X}^{(s)}_{MSPRSS}\right) < Var\left(\overline{X}^{(s)}_{RSS}\right),$$

where, $MSE\left(\overline{X}^{(s)}_{MPRSS}\right) = Var\left(\overline{X}^{(s)}_{MPRSS}\right) + Bias\left(\overline{X}^{(s)}_{MPRSS}\right)$

4. Comparison on Estimators

We can compare the three estimators for μ based on RSS, DPRSS and MPRSS procedure. For this purpose, we define the following relative precision (RP).

A. For RSS

$$RP' = \frac{Var\left(\overline{X}_{SRS}\right)}{Var\left(\overline{X}_{RSS}\right)}, \text{ for symmetric distribution}$$

$$= \frac{Var\left(\overline{X}_{SRS}\right)}{MSE\left(\overline{X}_{RSS}\right)}, \text{ for asymmetric distribution} \quad (11)$$

B. For PRSS

$$RP' = \frac{Var\left(\overline{X}_{SRS}\right)}{MSE\left(\overline{X}_{PRSS}\right)}, \text{ for symmetric distribution}$$

$$= \frac{Var\left(\overline{X}_{SRS}\right)}{MSE\left(\overline{X}_{PRSS}\right)}, \text{ for asymmetric distribution} \quad (12)$$

C. For DPRSS

$$RP'' = \frac{Var\left(\overline{X}_{SRS}\right)}{Var\left(\overline{X}_{DPRSS}\right)}, \text{ for symmetric distribution}$$

$$= \frac{Var\left(\bar{X}_{SRS}\right)}{MSE\left(\bar{X}_{DPRSS}\right)}$$, for asymmetric distribution (13)

D. For MSPRSS

$$RP'''' = \frac{Var(\bar{X}_{SRS})}{Var(\bar{X}_{MPRSS})}$$, for symmetric distribution

$$= \frac{Var(\bar{X}_{SRS})}{MSE(\bar{X}_{MSPRSS})}$$, for asymmetric distribution (14)

We have to examine RP for Symmetric and asymmetric distribution to know more about its efficiency. The Table 1 shows the RP of 5 symmetric and Table 2 gives the information of 5 asymmetric distribution for m = 6, 7, 11, 12 for each simulation 60,000 iteration are performed at p=25%.

From Tables 1 & 2, we get the following information

A. A gain in efficiency attained using MPRSS for estimation population mean for symmetric distribution, as example for m = 9 with s = 1, 2, 3 the relative efficiency of the MPRSS are 3.327, 10.276, 31.935 comparing it, with RSS is 2.812 for estimating population mean of a normal distribution with parameter 1 and 3. So for all symmetric distribution, the efficiency of $\bar{X}_{MSPRSS}$ is increasing in increase of stage s.

B. For asymmetric distribution, gain in efficiency is attained with smaller bias using MPRSS. For example, for Gamma distribution with parameter 1 and 3, the RP for MPRSS is 1.638, 2.399, 2.681 with biases 0.000, 0.701, 0.681 for m = 4 and s = 1, 2, 3 comparing it with RSS 1.593. So for asymmetric distribution, the efficiencies of $\bar{X}_{MSPRSS}$ is increasing in s on the converse of bias which is decreasing with s.

5. Sampling with Error in Ranking

In RSS, sampling mean is unbiased estimator of population mean without any proper information,

Table 1 : RP for RSS, PRSS, DPRSS and MSPRSS of 5 symmetrical disrtibution at p=25% with sample size 4, 5,9 and 10.

Symmetrical distribution	M	RSS	PRSS (s = 1)	DPRSS (s =2)	MSPRSS (s = 3)
		RP’	RP”	RP’”	RP””
UNIFORM (0,1)	4	2.500	3.125	27.018	364.084
	5	2.000	2.000	5.713	16.501
	9	3.000	2.562	9.401	33.264
	10	5.500	5.085	38.097	250.384
UNIFORM (0,2)	4	2.500	3.148	26.876	373.279
	5	2.000	2.000	5.773	16.041
	9	3.000	2.548	9.248	33.543
	10	5.500	5.128	38.466	250.729
NORMAL (0,1)	4	2.347	2.034	3.432	4.901
	5	1.914	1.914	3.295	4.998
	9	2.749	3.271	10.338	32.177
	10	4.787	5.736	31.288	160.948
NORMAL (1,3)	4	2.319	2.012	3.406	4.892
	5	1.910	1.910	3.296	5.143
	9	2.812	3.327	10.276	31.935
	10	4.779	5.850	31.721	162.267
LOGISTIC (-1,1)	4	2.229	1.706	1.904	2.017
	5	1.849	1.849	2.384	2.913
	9	2.563	3.637	11.437	33.681
	10	4.198	6.270	32.220	152.985

Table 2 : RP for RSS, PRSS, DPRSS and MSPRSS of 5 asymmetrical distribution at p=25% with sample size 4, 5,9 and 10.

Asymmetrical distribution	M	RSS	PRSS (s=1)		DPRSS(s = 2)		MPRSS(s=3)	
		RP	RP”	Bias	RP”’	Bias	RP””	Bias
Exponential(1)	4	1.636	1.636	0.000	1.394	0.230	1.678	0.551
	5	1.208	1.912	0.168	2.120	0.419	2.352	0.719
	9	2.177	2.607	0.151	5.579	0.130	14.248	0.085
	10	3.440	3.281	0.117	15.024	0.056	38.506	0.042
Exponential(2)	4	1.687	1.741	0.000	1.891	0.230	2.359	0.273
	5	1.162	1.922	0.168	2.119	0.719	2.352	0.119
	9	2.206	2.610	0.074	5.753	0.130	14.115	0.043
	10	3.426	3.288	0.059	14.916	0.028	38.725	0.021
Gamma(1,2)	4	1.655	1.695	0.000	2.733	0.078	2.681	0.184
	5	1.170	1.891	0.331	2.443	0.349	1.767	0.120
	9	2.240	2.622	0.300	5.675	0.258	14.230	0.170
	10	3.440	3.276	0.234	14.878	0.113	38.583	0.084
Gamma(1,3)	4	1.591.	1.638	0.000	2.399	0.701	2.681	0.687
	5	1.603	1.940	0.505	2.164	0.348	4.144	0.119
	9	2.161	2.638	0.449	5.670	0.389	14.279	0.256
	10	3.460	3..274	0.354	14.963	0.170	39.124	0.125
Weibull(1,3)	4	1.633	1.633	0.000	1.837	0.704	1.983	0.683
	5	1.154	1.937	0.505	2.160	0.349	4.152	0.117
	9	2.236	2.649	0.447	5.640	0.385	14.227	0.256
	10	3.471	3.245	0.352	14.808	0.170	38.509	0.126

whether it is perfect or imperfect. Hence,s it has smaller variance as compared with SRS having same sample size. Panda and Samantaray (2017) showed that DPRSS with error in ranking is unbiased estimator of population mean with assumption that, population is symmetric about mean. Hence, applying the above with MPRSS procedure in ranking with error may be defined as follows:

Let, $X^{(s)*}_{i(p(m+1);m)}$ and $X^{(s)*}_{i(q(m+1);m)}$ be the first and last multistage partitioned value of *i*th sample, (i = 1, 2, ..., m) having errors in ranking. Then using MSPRSS technique, the estimator of population mean with error in ranking can be represented as

$$\overline{X}^{(s)*}_{MSPRSSE_e} = \sum_{i=1}^{l} \overline{X}^{(s)*}_{i(p(m+1);m)} + \sum_{i=l+1}^{m} \overline{X}^{(s)*}_{i(q(m+1);m)} \quad (15)$$

where, l = (m/2)

$$\overline{X}^{(s)*}_{MSPRSSO_e} = \sum_{i=1}^{h} \overline{X}^{(s)*}_{i(p(m+1);m)} + \overline{X}^{(s)*}_{(h+1)(M;m)} + \sum_{i=h+1}^{m} \overline{X}^{(s)*}_{i(q(m+1);m)}, \quad (16)$$

where, h = ((m-1)/2)

The estimator of population mean in ranking with error having following properties:

a. $\overline{X}^{(s)}_{MSPRSSE_e}$ is an unbiased estimator of population mean with assumption that population is symmetric about its mean.

b. For symmetric distribution,

$$Var\left(\overline{X}^{(s)}_{MSPRSS_e}\right) < Var\left(\overline{X}^{(s)}_{SRS}\right)$$

and for asymmetric distribution,

$$MSE\left(\overline{X}^{(s)}_{MSPRSS_e}\right) < Var\left(\overline{X}^{(s)}_{SRS}\right)$$

The above properties can be proved on the basis of Takahasi and Wakimottto (1968), Dell and Clutter (1972), Muttalak (2003), Al-saleh and Al-kadiri (2000), Panda and Samantaray (2017).

6. Conclusion

In this article, it has been observed that, the estimator of proposed Multistage Partitioned Ranked Set Sampling is unbiased for population mean and is more efficient than SRS, RSS and DPRSS in case of symmetrical distribution. In case of asymmetric distribution, it also gives better result than its other sampling procedure with smaller Bias. Also from NPR analysis, it is found that there is greater efficiency in both symmetrical and asymmetrical distribution.

A. Appendix

Lemma A.1: $\overline{X}^{(s)}_{MSPRSS}$ is an unbiased estimator of the population mean, for given assumption that population is symmetric about its mean.

Proof : For *k*th cycle and *i*th sample,

- If m is even,

$$\left[x^{(s)}_{1(p(m+1))k} .x^{(s)}_{2(p(m+1))k} ....x^{(s)}_{\frac{m}{2}}\left(p(m+1)\right)k\right]$$

$$\left[x^{(s)}_{\frac{m}{2}+1}\left(q(m+1)\right)k ,x^{(s)}_{\frac{m}{2}+2}\left(q(m+1)\right)k ....\right.$$

$\left. x^{(s)}_{m}\left(q\ (m+1)\right)k\right]$ is the sample of size MPRSSE.

$$\Rightarrow \overline{X}^{(s)}_{MSPRSSE} = \frac{1}{m}\left[\sum_{i=1}^{l} X^{(s)}_{i(p(m+1))k} + \sum_{i=l+1}^{m} X^{(s)}_{i(q(m+1))k}\right]$$

$$E(\overline{X}^{(s)}_{MSPRSSE}) = \frac{1}{m}\left[\sum_{i=1}^{l} E(X^{(s)}_{i(p(m+1))k}) + \sum_{i=l+1}^{m} E(X^{(s)}_{i(q(m+1))k})\right]$$

$$= \frac{1}{m}\left(\frac{m}{2}\,.\,\mu^{(s)}{}_{(s;m)} + \frac{m}{2}\,\mu^{(s)}{}_{(m-s+1\,;\,m)}\right)$$

$$= \frac{1}{m}\times\frac{m}{2}\ \left(\mu^{(s)}{}_{s;m} + \mu^{(s)}{}_{(m-s+1\,;\,m)}\right)$$

$$= \frac{1}{2}\times 2\,\mu$$

- If m is odd,

$$\left[x^{(s)}_{1(p(m+1)k)}, x^{(s)}_{2(p(m+1)k)} \ldots, x^{(s)}_{\frac{m-1}{2}(p(m+1)k)}\right]\cdot\left[x^{(s)}_{\frac{m-1}{2}+1((m+1)/2)k}\right],$$

$$\left[x^{(s)}_{\frac{m-1}{2}+2(q(m+1))k}, \ldots, x^{(s)}_{\frac{m-1}{2}+3(q(m+1)k)} \ldots, x^{(s)}_{m(q(m+1))k}\right] \text{ is}$$

the sample of size MPRSSO.

$$\Rightarrow (\overline{X}^{(s)}_{MSPRSSO}) = \frac{1}{m}\left[\sum_{i=1}^{h} (X^{(s)}_{i(p(m+1))k})\right.$$

$$\left. + \sum_{i=h+2}^{m} E(X^{(s)}_{i(q(m+1))k}) + E(X^{(s)}_{(h+1)(M;m)})\right]$$

$$\Rightarrow E(\overline{X}^{(s)}_{MSPRSSO}) = \frac{1}{m}\left[\sum_{i=1}^{l} E(X^{(s)}_{i(p(m+1))k})\right.$$

$$\left. + \sum_{i=l+1}^{m} E(X^{(s)}_{i(q(m+1))k}) + E(X^{(s)}_{(h+1)(M;m)})\right]$$

$$= \frac{1}{m}\left[\frac{m-1}{2}\left(\mu^{(s)}{}_{s;m} + \mu^{(s)}{}_{m-s+1;m}\right) + \mu\right] = \mu$$

Hence, $\overline{X}^{(s)}_{MSPRSS}$ is an unbiased estimator of the population mean.

Lemma A.2: $Var\left[\overline{X}^{(s)}_{MSPRSS}\right]$ is less than each of $Var\left(\overline{X}_{SRS}\right)$ and $Var\left(\overline{X}_{RSS}\right)$.

Proof: For m is even,

Then Variance will be

$$\Rightarrow \text{var}\left(\overline{X}^{(s)}_{MSPRSSE}\right) = \frac{1}{m^2}\left(\sum_{i=1}^{m/2} \text{var}\left(X^{(s)}{}_{i(s\,;m)}\right)\right.$$

$$\left. + \frac{1}{m^2}\sum_{j=\frac{m}{2}+1}^{m} \text{var}\left(X^{(s)}{}_{i\ (m-s+1\,;m)}\right)\right)$$

$$= \frac{1}{2m}\left(\sigma^{(s)^2}{}_{s\,;\,m} + \sigma^{(s)^2}{}_{m-s+1;m}\right) = \frac{\sigma^{(s)2}_{s,m}}{m}$$

$$\Rightarrow var\left(MSPRSSE\right) < var\left(RSS\right) < var\left(SRS\right)$$

For m is odd,

Then Variance can be defined as

$$\Rightarrow \text{var}\left(\overline{X}^{(s)}_{MSPRSSO}\right) = \frac{1}{m}\left(\sum_{i=1}^{m-1/2} \text{var}\ \left(X^{(s)}{}_{i(s;m}\right)\right.$$

$$\left. + \text{var}\left(X^{(s)}{}_{\frac{m+1}{2}\,.\ m}\right) + \sum_{i\ \frac{m+1}{2}}^{m} \text{var}\left(X^{(s)}{}_{j(m-s+1;m)}\right)\right)$$

$$= \frac{1}{m^2}\left(\frac{m-1}{2}\left(\sigma^{(s)^2}{}_{s;m} + \sigma^{(s)^2}\,m-s+1;m\right) + \sigma^{(s)^2}{}_{\frac{m+1}{2}\,;\,m}\right)$$

$$= \frac{1}{m^2}\left(\frac{m-1}{2}(2\sigma^{(s)2}_{s,m}) + \sigma^{(s)2}_{s,m}\right)$$

$$= \frac{1}{m}\sigma^{(s)2}_{s,m}$$

$$\Rightarrow \text{var}(MSPRSSO) < \text{var}(RSS) < \text{var}(SRS)$$

because, $\frac{\sigma_{s,m}^{(s)2}}{m} < \frac{\sigma_{s,m}^{2}}{m} < \frac{\sigma^2}{m}$

Acknowledgement

Authors are highly thankful to the Editor-in-Chief and Anonymous reviewer for their constructive and fruitful comments which improved much the earlier version on this manuscript.

References

Al-Saleh, M. F. and M. Al-Kadiri (2000). Double ranked Set Sampling. *Statist. Probab. Lett.*, **48**, 205-212.

Al-Saleh, M.F. and A. Al-Omari (2002). Multistage Ranked Set Sampling. *Journal of Statistical Planning and Inference, 102(2),* 273-286.

Dell, T. R. and J. L. Clutter (1972). Ranked Set Sampling theory with order statistics background. *Biometrics*, **28**, 545-555.

Jemain, A. A., A. Al-Omari and K. Ibrahim (2007). Multistage Median Ranked Set Sampling for estimating population median. *Journal of Mathematics and Statistics,* **3(2)**, 58-64.

McIntyre, G.A. (1952). A method of unbiased selective sampling, using Ranked sets. *Australian Journal of Agriculture Research*, **3**, 385-390.

Muttalak, H. A. (1997). Median Ranked set Sampling. *Journal of Applied Statistical Science*, **6**, 245-255.

Panda, K. B. and M. Samantaray (2017). Double Partitioned Ranked Set Sampling: An Efficient Estimation Technique. *Int. J. for research in App. Sc. and Engineering Tech*., **5(X)**, 2231-2245

Panda, K. B. and M. Samantaray (2017). Partitioned Ranked Set Sampling: Introduction to a Efficient Estimation Technique. *Int. J. of Scientific Research and Reviews*, Oct-Dec, Issue 2017, 181-190.

Strokes, S. L. (1995). Parametric ranked set sampling. *Annals of the Institute of Statistical Mathematics*, **47**, 465- 482.

Takahasi, K. and K. Wakimoto (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the institute of Statistical Mathematics*, **20**, 1-31.